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The mass transfer and stability in systems with large concentration gradients—I. Mass transfer kinetics

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Abstract—The theoretical analysis about the influence of high concentration and large concentration gradients on the hydrodynamics and mass transfer in the approximations of the laminar boundary layer has been done. The results obtained show that the change in the density with the concentration influences the hydrodynamics in gases and liquids and does not influence the mass transfer in gases. The change in the viscosity with the concentration influences the hydrodynamics in gases and liquids and the mass transfer. The change in the diffusivity with the concentration does not influence the hydrodynamics and the mass transfer. It has been clearly seen that the non-linear theory on mass transfer at the constant values of density, viscosity and diffusivity has enough accuracy for gases and liquids, if the density of transferred substance is not sufficiently different from the density of the gas mixture.

INTRODUCTION

The influence of the large concentration gradients on the mass transfer kinetics and hydrodynamic stability of systems with intensive interphase mass transfer has been investigated in a number of papers [1–8]. It has been shown that under these conditions secondary flows are induced directed normally to phase boundary. They change the mass transfer rate and the critical Reynolds numbers for the transition from laminar to turbulent flow in the boundary layer. All these results were obtained assuming that the density, the viscosity and the diffusivity do not depend on the concentration of the transferred substance, i.e. only the effect of large concentration gradient was researched. Examining these cases one can find that the concentration of the transferred substance can be significantly high and this should lead to an additional effect, i.e. to the combined effect due to the effect of high concentrations and the effect of the large concentration gradients.

MATHEMATICAL MODEL

The mathematical model considers mass transfer in the boundary layer in the case of a stream flow along a semi-infinite plate, without limitations to concentration and its gradient. Under these conditions the mathematical model takes the following form [9]:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0$$

$$\rho\left(u\frac{\partial c}{\partial x}+v\frac{\partial c}{\partial y}\right)=\frac{\partial}{\partial y}\left(\rho D\frac{\partial c}{\partial y}\right)$$

$$x=0 \quad u=u_0 \quad c=c_0$$

$$y=0 \quad u=0 \quad v=-\frac{MD\rho^*}{\rho_0^*}\frac{\partial}{\partial y}\left(\frac{c}{\rho}\right) \quad c=c^*$$

$$y\to\infty \quad u=u_0 \quad c=c_0 \qquad (1)$$

where $\rho^*=\rho_0^*+Mc^*$ and ρ, μ, D depend on the concentration:

$$\rho=\rho(c) \quad \mu=\mu(c) \quad D=D(c). \qquad (2)$$

The mass transfer rate can be expressed by the mass transfer coefficient. We will define this rate from the average diffusion flux, trough surface with the specific length l:

$$J=k(c^*-c_0)=\frac{1}{l}\int_0^l D\left(\frac{\partial c}{\partial y}\right)_{y=0}\mathrm{d}x. \qquad (3)$$

The thickness of diffusion boundary layer in gases and liquids is of different order of magnitude. That is why we use different numerical algorithms.

MASS TRANSFER IN GASES

The thickness of laminar and diffusion boundary layers in gases is of the same order of magnitude, so one characteristic scale can be applied:

$$\delta_0=\sqrt{\frac{D_0 l}{u_0}}. \qquad (4)$$

NOMENCLATURE

c	concentration	x	coordinate
D	diffusivity	y	coordinate.
J	mass transfer rate		
k	mass transfer coefficient	Greek symbols	
l	specific length of the interface	μ	viscosity [N.s m^{-3}]
Sc	Schmidt number	ρ	density [kg m^{-3}].
Sh	Sherwood number		
Pe	Peclet number	Indices	
u	velocity in x-direction	0	initial values
v	velocity in y-direction	*	on the interface.

The problem (1) can be expressed in the terms of the following dimensionless variables:

$$x = lX \quad y = \delta_0 Y$$

$$u = u_0 U \quad v = u_0 \frac{\delta_0}{l} V \quad c = c_0 + (c^* - c_0)C. \quad (5)$$

Introducing equation (5) into equation (1) leads to the following equations

$$\phi\left(U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = Sc\frac{\partial}{\partial Y}\left(\psi\frac{\partial U}{\partial Y}\right)$$

$$\frac{\partial}{\partial X}(\phi U) + \frac{\partial}{\partial Y}(\phi V) = 0$$

$$\phi\left(U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y}\right) = \frac{\partial}{\partial Y}\left(\phi\omega\frac{\partial C}{\partial Y}\right)$$

$$X = 0 \quad U = 1 \quad C = 0$$

$$Y = 0 \quad U = 0 \quad V = -\theta_0 \frac{\partial}{\partial Y}\left(\frac{\frac{c_0}{\Delta c} + C}{\phi}\right) \quad C = 1$$

$$Y \to \infty \quad U = 1 \quad C = 0 \quad (6)$$

where

$$\theta_0 = \frac{M\Delta c_0}{\rho_0^*}\phi(1)\omega(1) \quad \rho^* = \rho_0\phi(1)$$

$$\rho_0^* = \rho_0\phi(1) - Mc^*$$

$$\Delta c_0 = c^* - c_0 \quad Sc = \frac{\mu_0}{\rho_0 D_0}$$

$$\phi = \phi(C) = \rho/\rho_0 \quad \psi = \psi(C) = \mu/\mu_0$$

$$\omega = \omega(C) = D/D_0$$

$$\phi(0) = 1 \quad \psi(0) = 1 \quad \omega(0) = 1. \quad (7)$$

The solution of the problem (6) can be obtained after introducing the similarity variables:

$$\phi U = \Phi' \quad \phi V = \frac{1}{2\sqrt{X}}(\Phi'\eta - \Phi) \quad C = F$$

$$\Phi = \Phi(\eta) \quad F = F(\eta) \quad \eta = \frac{Y}{\sqrt{X}} \quad \Phi' = \frac{d\Phi}{d\eta}.$$

Hence, directly from equation (6) we can obtain the following:

$$2Sc\phi^2\psi\Phi''' + \phi^2\Phi\Phi'' - \phi\phi'\Phi\Phi'F' + 2Sc\phi(\phi\psi' - \phi'\psi)\Phi''F' - 2Sc\phi'(\phi\psi' - 2\phi'\psi)\Phi'F'^2 = 0$$

$$2\phi\omega F'' + 2(\phi'\omega + \phi'\psi)F'^2 + \Phi F' = 0$$

$$\Phi(0) = -\theta F'(0) \quad \Phi'(0) = 0 \quad \Phi'(\infty) = 1$$

$$F(0) = 1 \quad F(\infty) = 0 \quad \theta = 2\theta_0 \frac{\Delta c_0 \phi(1) - c^*\phi'(1)}{\Delta c_0 \phi(1)}. \quad (9)$$

The functions ϕ, ψ and ω in equation (9) are set outwardly by spline approximations of experimental dependencies of ρ, μ and D from c. For a wide range of gas mixtures these functions can be obtained with enough accuracy through a linear approximation

$$\phi = 1 + \bar{\rho}C \quad \psi = 1 + \bar{\mu}C \quad \omega = 1 + \bar{D}C. \quad (10)$$

The introduction of equation (10) into equation (9) leads to following equations

$$2Sc(1+\bar{\rho}F)^2(1+\bar{\mu}F)\Phi''' + (1+\bar{\rho}F)^2\Phi\Phi'' - \bar{\rho}(1+\bar{\rho}F)\Phi\Phi'F' + 2Sc(1+\bar{\rho}F)[\bar{\mu}(1+\bar{\rho}F) - \bar{\rho}(1+\bar{\mu}F)]\Phi''F' - 2Sc\bar{\rho}[\bar{\mu}(1+\bar{\rho}F) - 2\bar{\rho}(1+\bar{\mu}F)]\Phi'F'^2 = 0$$

$$2(1+\bar{\rho}F)(1+\bar{D}F)F'' + 2[\bar{\rho}(1+\bar{D}F) + \bar{D}(1+\bar{\rho}F)]F'^2 + \Phi F' = 0$$

$$\theta = 2\theta_0 \frac{1 - \frac{c_0}{\Delta c_0}\bar{\rho}}{1+\bar{\rho}}. \quad (11)$$

The parameters $\bar{\rho}$ and $\bar{\mu}$ in equation (11) are small, while $\bar{D}=0$. Omitting the square terms regarding small parameters $\bar{\rho}$ and $\bar{\mu}$ leads to:

$$2Sc(1+2\bar{\rho}F+\bar{\mu}F)\Phi'''+(1+2\bar{\rho}F)\Phi\Phi''$$
$$-\bar{\rho}\Phi\Phi'F'+2Sc(\bar{\mu}-\bar{\rho})\Phi''F'=0$$
$$2(1+\bar{\rho}F)F''+2\bar{\rho}F'^2+\Phi F'=0$$
$$\Phi(0)=-\theta F'(0)\quad \Phi'(0)=0\quad \Phi'(\infty)=1$$
$$F(0)=1\quad F(\infty)=0. \qquad (12)$$

The problem (12) can be solved conveniently using the following algorithm:

(1) Determination of the zeroth approximations of Φ and F by solving the boundary problem:

$$2\Phi'''^{(0)}+\Phi^{(0)}\Phi''^{(0)}=0$$
$$\Phi^{(0)}(0)=0\quad \Phi'^{(0)}(0)=0\quad \Phi''^{(0)}(0)=0.33206$$
$$(\Phi'^{(0)}(\infty)=1)$$
$$2F''^{(0)}+\Phi^{(0)}F'^{(0)}=0$$
$$F^{(0)}(0)=1\quad F'^{(0)}(0)=0.33206\quad (F^{(0)}(\infty)=0). \qquad (13)$$

(2) Determining Φ at the kth iteration:

$$2Sc(1+2\bar{\rho}F^{(k-1)}+\bar{\mu}F^{(k-1)})\Phi'''^{(k)}$$
$$+(1+2\bar{\rho}F^{(k-1)})\Phi^{(k)}\Phi''^{(k)}$$
$$-\bar{\rho}\Phi^{(k-1)}\Phi'^{(k-1)}F'^{(k-1)}+2Sc(\bar{\mu}-\bar{\rho})\Phi''^{(k-1)}F'^{(k-1)}=0$$
$$\Phi^{(k)}(0)=-\theta F'^{(k-1)}(0)\quad \Phi'^{(k)}(0)=0\quad \Phi'^{(k)}(\infty)=1 \qquad (14)$$

while the value of $\Phi'^{(k)}(0)$ is varied till the condition $\Phi''^{(k)}(6)=1$ is reached with accuracy 10^{-3}.

(3) Determining F at the kth iteration

$$2(1+\bar{\rho}F^{(k-1)})F''^{(k)}+2(\bar{\rho}F'^{(k-1)})^2+\Phi^{(k)}F'^{(k)}=0$$
$$F^{(k)}(0)=1\quad F^{(k)}(\infty)=0 \qquad (15)$$

while $F'^{(k)}(0)$ is varied till $F^{(k)}(0)=0$ with the accuracy 10^{-3}.

(4) The calculation procedure (from step 2 of the algorithm) is repeated until a result confirming:

$$|\Phi''^{(k)}(0)-\Phi''^{(k-1)}(0)|\leqslant 10^{-3}$$
$$|F'^{(k)}(0)-F'^{(k-1)}(0)|\leqslant 10^{-3} \qquad (16)$$

is obtained.

The integration of equations (13)–(15) is done numerically with a step $h=10^{-2}$ in the interval $0\leqslant\eta\leqslant 6$.

The results for $\Phi''(0)$ and $F(0)$ in the case of $Sc=1$ are shown in Table 1. for different values of θ, $\bar{\rho}$ and $\bar{\mu}$. They are obtained by 3–4 iterations. The mass transfer rate in gases can be determined from data in Table 1. In order to do this equations (5) and (8) are introduced into equation (3):

$$Sh=\frac{kl}{D_0}=2Pe^{1/2}F'(0)\quad Re=\frac{u_0 l}{D_0}. \qquad (17)$$

Table 1. Comparison data for the momentum transfer ($\Phi''(0)$) and the mass transfer ($F'(0)$) at the high concentrations (effects due to density ($\bar{\rho}\neq 0$) and viscosity ($\bar{\mu}\neq 0$)) and large concentration gradients ($\theta\neq 0$) in gases

No.	θ	$\bar{\rho}$	$Sc=1$ $\bar{\mu}$	$\Phi''(0)$	$-F'(0)$
1	0	0	0	0.332	0.332
2	0.3	0	0	0.301	0.299
3	−0.3	0	0	0.373	0.372
4	0.3	0.15	0	0.356	0.187
5	0	0.15	0	0.379	0.198
6	−0.3	−0.15	0	0.329	0.531
7	0.3	0	0.2	0.264	0.292
8	0	0	0.2	0.290	0.322
9	−0.3	0	−0.2	0.447	0.386
10	0.3	0.15	0.2	0.320	0.187
11	0	0.15	0.2	0.340	0.198
12	−0.3	0.15	0.2	0.362	0.211
13	0	−0.15	0	0.280	0.446
14	0	0	−0.2	0.394	0.343
15	0	−0.15	−0.2	0.347	0.469
16	−0.3	−0.15	−0.2	0.417	0.558

The results obtained in Table 1 show that (Fig. 1) the dependence of $\Phi''(0)$ and $F''(0)$ from θ, $\bar{\rho}$ and $\bar{\mu}$ is monotone. The change in viscosity, $\bar{\mu}$, practically does not influence the mass transfer rate ($F'(0)$), while the influence of the density $\bar{\rho}$ is 6–7 times greater than the non-linear mass transfer (θ).

MASS TRANSFER IN LIQUIDS

The thickness of laminar and diffusion boundary layers in liquids are of different order of magnitude, so two specific scales should be taken into account:

$$\delta_1=\sqrt{\frac{\mu_0 l}{\rho_0 u_0}}\quad \delta_2=\sqrt{\frac{D_0 l}{u_0}}\quad \frac{\delta_1}{\delta_2}=\varepsilon=Sc^{1/2}. \qquad (18)$$

Considering these two scales, the following dimensionless variables should be introduced

$$x=lX\quad y=\delta_1 Y_1=\delta_2 Y_2$$
$$u=u_0U_1(X,Y_1)=u_0U_2(X,Y_2)$$
$$v=u_0\frac{\delta_1}{l}V_1(X,Y_1)=u_0\frac{\delta_2}{l}V_2(X,Y_2)$$
$$c=c_0+\Delta c_0C_1(X,Y_1)=c_0+\Delta c_0C_2(X,Y_2) \qquad (19)$$

where

$$Y_2=\varepsilon Y_1$$
$$U_2(X,Y_2)=U_1(X,\varepsilon^{-1}Y_2)\quad U_1(X,Y_1)=U_2(X,\varepsilon Y_1)$$

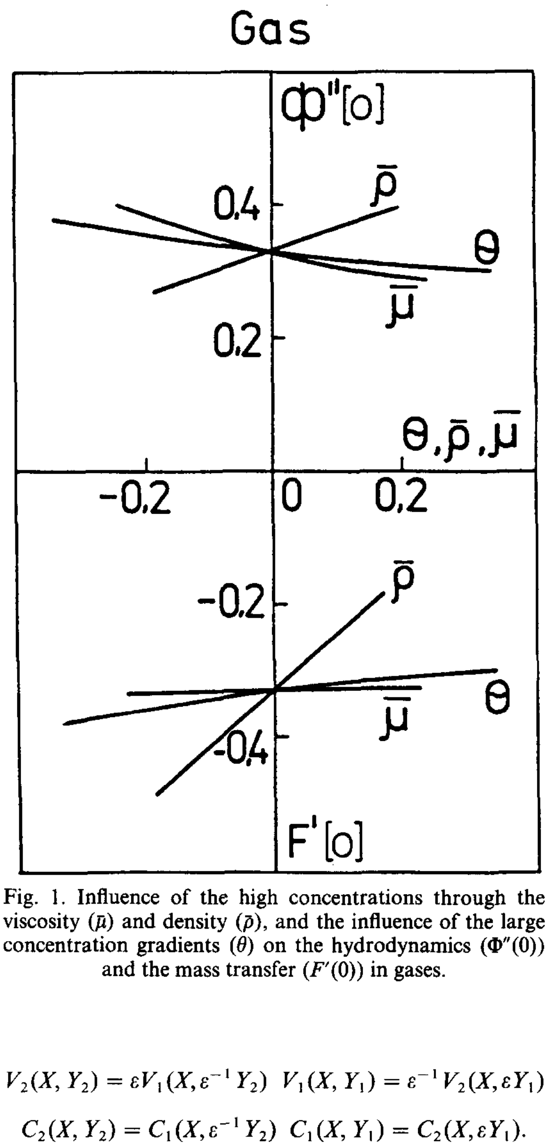

Fig. 1. Influence of the high concentrations through the viscosity ($\bar{\mu}$) and density ($\bar{\rho}$), and the influence of the large concentration gradients (θ) on the hydrodynamics ($\Phi''(0)$) and the mass transfer ($F'(0)$) in gases.

$$V_2(X, Y_2) = \varepsilon V_1(X, \varepsilon^{-1} Y_2) \quad V_1(X, Y_1) = \varepsilon^{-1} V_2(X, \varepsilon Y_1)$$

$$C_2(X, Y_2) = C_1(X, \varepsilon^{-1} Y_2) \quad C_1(X, Y_1) = C_2(X, \varepsilon Y_1). \tag{20}$$

In new variables the problem obtains the following form:

$$\phi_1\left(U_1 \frac{\partial U_1}{\partial X} + V_1 \frac{\partial U_1}{\partial Y_1}\right) = \frac{\partial}{\partial Y_1}\left(\psi_1 \frac{\partial U_1}{\partial Y_1}\right)$$

$$\frac{\partial}{\partial X}(\phi_1 U_1) + \frac{\partial}{\partial Y_1}(\phi_1 V_1) = 0$$

$$\phi_2\left(U_2 \frac{\partial C_2}{\partial X} + V_2 \frac{\partial C_2}{\partial Y_2}\right) = \frac{\partial}{\partial Y_2}\left(\phi_2 \omega_2 \frac{\partial C_2}{\partial Y_2}\right)$$

$$X_1 = 0 \quad U_1 = U_2 = 1 \quad C_1 = C_1 = 0$$

$$Y_1 = Y_2 = 0 \quad U_1 = U_2 = 0 \quad C_1 = C_2 = 0$$

$$V_2 = -\theta_0 \frac{\partial}{\partial Y_2}\left(\frac{\frac{c_0}{\Delta c} + C_2}{\phi_2}\right)$$

$$Y_1 = Y_2 \to \infty \quad U_1 = U_2 = 1 \quad C_1 = C_2 = 0. \tag{21}$$

The boundary problem can be expressed by the following similarity variables:

$$\phi_1 U_1 = \Phi_1'(\eta_1) \quad \phi_2 U_2 = \Phi_2'(\eta_2)$$

$$\eta_1 = \frac{Y_1}{\sqrt{X}} \quad \eta_2 = \frac{Y_2}{\sqrt{X}}$$

$$\phi_1 V_1 = \frac{1}{2\sqrt{X}}(\Phi_1' \eta_1 - \Phi_1) \quad \phi_2 V_2 = \frac{1}{2\sqrt{X}}(\Phi_2' \eta_2 - \Phi_2)$$

$$C_1 = F_1(\eta_1) \quad C_2 = F_2(\eta_2) \quad \eta_2 = \varepsilon \eta_1 \tag{22}$$

while for ϕ, ψ and ω we can use linear approximations:

$$\phi_i = 1 + \bar{\rho} F_i \quad \psi_i = 1 + \bar{\mu} F_i \quad \omega_i = 1 + \bar{D} F_i \quad i = 1, 2. \tag{23}$$

In new variables equation (22) obtains the following form:

$$2(1 + 2\bar{\rho} F_1 + \bar{\mu} F_1)\Phi_1''' + (1 + 2\bar{\rho} F_1)\Phi_1 \Phi_1''$$
$$- \bar{\rho}\Phi_1 \Phi_1' F_1' + 2(\bar{\mu} - \bar{\rho})\Phi_1' F_1' = 0$$
$$2(1 + \bar{\rho} F_2 + \bar{D} F_2) F_2'' + 2(\bar{\rho} + \bar{D}) F_2'^2 + \Phi_2 F_2' = 0$$
$$\Phi_2(0) = -\theta F_2'(0) \quad \Phi_1'(0) = 0 \quad \Phi_1'(\infty) = 1$$
$$F_2(0) = 1 \quad F_2(\infty) = 0 \tag{24}$$

where

$$F_1(\eta_1) = F_2(\eta_2) = F_2(\varepsilon \eta_1) \quad F_1'(\eta_1) = \varepsilon F_2'(\varepsilon \eta_1)$$
$$\Phi_2(\eta_2) = \varepsilon \Phi_1(\eta_1) = \varepsilon \Phi_1(\varepsilon^{-1} \eta_2)$$
$$\Phi_2(0) = \varepsilon \Phi_1(0) = -\theta F_2'(0). \tag{25}$$

The problem (25) can be directly solved using the following algorithm:

(1) Determination of the zeroth approximations of $\Phi_1(\eta_1)$ by integration of the equation:

$$\Phi_1'''^{(0)} + \Phi_1^{(0)} \Phi_1''^{(0)} = 0$$
$$\Phi_1^{(0)}(0) = 0 \quad \Phi_1'^{(0)}(0) = 0 \quad \Phi_1'^{(0)}(\infty) = 1 \tag{26}$$

with a step $h_1 = 0.06/\varepsilon$ in the interval $0 \leqslant \eta_1 \leqslant 6$. We should vary $\Phi_1'^{(0)}$ until condition $\Phi_{(0)}'^{(0)} \geqslant 0.999$ is satisfied.

(2) Determination of the zeroth approximations of $\Phi_2(\eta_2)$:

$$\Phi_2^{(0)}(\eta_2) = \varepsilon \Phi_1^{(0)}(\eta_1) \quad \eta_2 = \varepsilon \eta_1 \quad 0 \leqslant \eta_1 \leqslant 6. \tag{27}$$

(3) Determination of the zeroth approximations of $F_2(\eta_2)$ by integration of the equation.

$$F_2''^{(0)} + \Phi_2^{(0)} F_2'^{(0)} = 0 \quad F_2^{(0)}(0) = 1 \quad F_2^{(0)}(\infty) = 0 \tag{28}$$

with a step $h_2 = 0.06$ in the interval $0 \leqslant \eta_2 \leqslant 60$. In order to do this $F_2'^{(0)}(0)$ is varied until condition $F_{2(0)}^{(0)} \leqslant 0.001$ is satisfied.

(4) Determination of the zeroth approximations of $F_1(\eta_1)$ and $F_1'(\eta_1)$:

$$F_1^{(0)}(\eta_1) = F_2^{(0)}(\eta_2) = F_2^{(0)}(\varepsilon\eta_1)$$

$$F_1'^{(0)}(\eta_1) = \varepsilon F_2'^{(0)}(\eta_2) = \varepsilon F_2'^{(0)}(\varepsilon\eta_1). \quad (29)$$

(5) Determining $\Phi_1(\eta_1)$ at the kth iteration:

$$2(1+2\bar{\rho}F_1^{(k-1)}+\bar{\mu}F_1^{(k-1)})\Phi_1'''^{(k)} + (1+2\bar{\rho}F_1^{(k-1)})\Phi_1^{(k)}\Phi_1''^{(k)} - \bar{\rho}\Phi_1^{(k-1)}\Phi_1'^{(k-1)}F_1'^{(k-1)} + 2(\bar{\mu}-\bar{\rho})\Phi_1''^{(k-1)}F_1'^{(k-1)} = 0$$

$$\Phi_1^{(k)}(0) = -\frac{\theta}{\varepsilon}F_2'^{(k-1)}(0)$$

$$\Phi_1'^{(k)}(0) = 0 \quad \Phi_1'^{(k)}(\infty) = 1 \quad (30)$$

while the value of $\Phi_1''^{(k)}(0)$ is varied till the condition $\Phi_1'^{(k)}(6) \geqslant 0.999$ is reached.

(6) Determining $\Phi_2(\eta_2)$ at the kth iteration:

$$\Phi_2^{(k)}(\eta_2) = \varepsilon\Phi_1^{(k)}(\eta_1) = \varepsilon\Phi_1^{(k)}(\varepsilon^{-1}\eta_2) \quad 0 \leqslant \eta_2 \leqslant 60. \quad (31)$$

(7) Determining $F_2(\eta_2)$ at the kth iteration with a step h_2 in the interval $0 \leqslant \eta_2 \leqslant 60$:

$$2(1+\bar{\rho}F_2^{(k-1)}+\bar{D}F_2^{(k-1)})F_2''^{(k)} + 2(\bar{\rho}+\bar{D})(F_2'^{(k-1)})^2 + \Phi_2^{(k)}F_2'^{(k)} = 0$$

$$F_2^{(k)}(0) = 1 \quad F_2^{(k)}(\infty) = 0 \quad (32)$$

while the value of $F_2'^{(k)}(0)$ is varied till the condition $F_1^{(k)}(6) \leqslant 0.001$ is satisfied.

(8) Determining $F_1(\eta_1)$ and $F_1'(\eta_1)$ at the kth iteration:

$$F_1^{(k)}(\eta_1) = F_2^{(k)}(\eta_2) = F_2^{(k)}(\varepsilon\eta_1)$$

$$F_1'^{(k)}(\eta_1) = \varepsilon F_2'^{(k)}(\eta_2) = \varepsilon F_2'^{(k)}(\varepsilon\eta_1)$$

$$0 \leqslant \eta_1 \leqslant 6. \quad (33)$$

(9) The calculation procedure (from step 5 of the algorithm) is repeated until result confirming:

$$|\Phi_1''^{(k)}(0) - \Phi_1''^{(k-1)}(0)| \leqslant 10^{-3}$$

$$|F_2'^{(k)}(0) - F_2'^{(k-1)}(0)| \leqslant 10^{-3} \quad (34)$$

is obtained.

The results obtained for $\Phi_1''(0)$ and $F_2'(0)$ at $\varepsilon = 10$ and for different values of θ, $\bar{\rho}$, $\bar{\mu}$ and $\bar{D}$ are shown in Table 2. They are obtained with 3–4 iterations. The mass transfer rate in liquids can be determined from data in Table 2. In order to do this equations (20) and (23) are introduced into equation (3):

$$Sh = 2(1+\bar{D})Pe^{1/2}F_2'(0). \quad (35)$$

The results obtained in Table 2 show (Fig. 2) that

Table 2. Comparison data for the momentum transfer ($\Phi''(0)$) and the mass transfer ($F'(0)$) at the high concentrations (effects due to density ($\bar{\rho} \neq 0$), viscosity ($\bar{\mu} \neq 0$) and diffusivity ($\bar{D} \neq 0$)) and large concentration gradients ($\theta \neq 0$) in liquids

			$Sc = 100$			
No.	θ	$\bar{\rho}$	$\bar{\mu}$	$\bar{D}$	$\Phi''(0)$	$-F'(0)$
1	0	0	0	0	0.332	0.332
2	0.03	0	0	0	0.330	0.176
3	−0.03	0	0	0	0.334	0.206
4	0	0.15	0	0	0.397	0.194
5	0	−0.15	0	0	0.201	0.181
6	0	0	0.2	0	0.272	0.186
7	0	0	−0.2	0	0.418	0.194
8	0	0	0	0.30	0.332	0.192
9	0	0	0	−0.30	0.332	0.186
10	0.03	0.15	0.2	0.30	0.272	0.177
11	−0.03	0.15	0.2	0.30	0.275	0.200
12	0.03	−0.15	−0.2	−0.30	0.243	0.164
13	−0.03	−0.15	−0.2	−0.30	0.247	0.206
14	0.3	0	0	0	0.318	0.135
15	−0.1	0	0	0	0.342	0.268

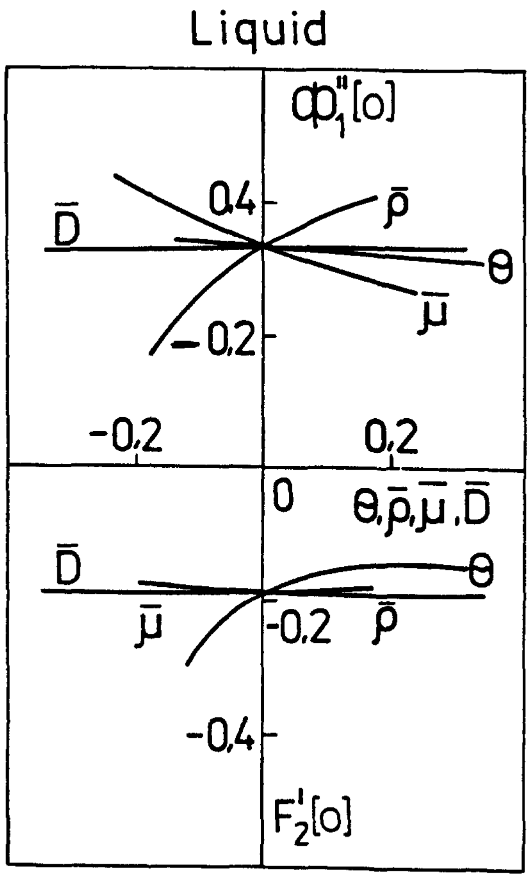

Fig. 2. Influence of the high concentrations through the viscosity ($\bar{\mu}$) and density ($\bar{\rho}$) and diffusivity ($\bar{D}$), and the influence of the large concentration gradients (θ) on the hydrodynamics ($\Phi''(0)$) and the mass transfer ($F'(0)$) in liquids.

the influence of density $\bar{\rho}$ and viscosity $\bar{\mu}$ on the hydrodynamics ($\Phi_1''(0)$) is analogous to that one in the case of gases, while this influence on the mass transfer rate ($F_2'(0)$) is practically insignificant. The change in diffusivity $\bar{D}$ does not affect $\Phi_1''(0)$ as well as $F_2'(0)$.

RESULTS AND DISCUSSIONS

The theoretical analysis of the influence of high concentration gradients of transferred substance on the hydrodynamics ($\Phi''(0)$) and mass transfer ($F'(0)$) through the concentration dependencies of density ($\bar{\rho}$), viscosity ($\bar{\mu}$) and diffusivity ($\bar{D}$) show that:

- the change in the density with the concentration influences the hydrodynamics in gases and liquids and does not influence the mass transfer in gases;
- the change in the viscosity with the concentration influences the hydrodynamics in gases and liquids and the mass transfer;
- the change in the diffusivity with the concentration does not influence the hydrodynamics and the mass transfer.

These results show that the non-linear theory on mass transfer at the constant values of density, viscosity and diffusivity [1–7] has enough accuracy for gases and liquids if the density of transferred substance is not sufficiently different from the density of the gas mixture. That is why we can considerably simplify the models of mass transfer in systems with intensive interphase mass transfer. The data for the velocity field in the laminar boundary layer give the opportunity to analyze the hydrodynamic stability of the flow.

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